Table 2. Values of n and the maximum error of the correlation equation (29) for the whole time domain

Range of Pr	n	Maximum error (%)
Step change in temperature:		
$0.01 \leqslant Pr \leqslant 0.6$	2.09	7.41
$0.7 \leqslant Pr \leqslant 10\,000$	3.68	6.52
Step change in heat flux:		
$0.01 \leqslant Pr \leqslant 0.6$	2.21	7.65
$0.7 \leqslant Pr \leqslant 10\,000$	3.85	5.92

transient conduction and the boundary-layer thickness of steady forced convection. With the growing boundary-layer thickness as a basis, we have introduced a nondimensional transverse coordinate and a dimensionless time, which are with proper scales for the initial, the transition, and the final stages of unsteady convection. Consequently, a comprehensive formulation and precise numerical solutions can be obtained over the entire transient history including unsteady conduction, true unsteady convection, and steady convection. Moreover, comprehensive and accurate correlation equations of Nusselt numbers have been developed, which are based on the solutions of unsteady conduction and steady convection.

The proposed method has been proved to be very effective and accurate via the demonstration of unsteady forced convection of a rotating disk. The present solution method has been applied successfully to many other unsteady convection heat transfer problems of various configurations and fluids.

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A unified correlation of laminar convective heat transfer from hot and cold circular cylinders in a uniform air flow

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1. INTRODUCTION

The heat transfer on a circular cylinder embedded in a uniform cross flow is important not only due to its fundamental nature *per se* but also due to related engineering applications. Despite the simplicity of the relevant geometry, the flow over a cylinder frequently entails multi-faceted flow structures such as laminar boundary layer, transition, turbulent boundary layer, separation and wake formation. Fortunately, for the range of the Reynolds number pertinent to the hot-wire anemometry (commonly $Re \leqslant 40$), the flow is known to be steady and laminar. In this range of Re, the Nusselt number (dimensionless heat transfer coefficient) has been expressed as a canonical function of Pr and Re, with empirical correction factors accounting for the variation of fluid properties. Particularly for the case of air, the available heat transfer correlations are of the following form:

$$Nu = (A + B\,Re^n)(T_m/T_a)^p \tag{1}$$

due to a very weak dependency of Pr on temperature. However, a use of the mean temperature T_m in equation (1) is traditional rather than physically justifiable. A survey of the literature indicates that the existing correlations are of limited applicability to a narrow range of temperature ratios, typically for T_m/T_a smaller than 1.2. Furthermore, the available correlations are all biased, i.e. valid only for either hot ($T_w > T_a$) or cold ($T_w < T_a$) cylinders. Therefore, such hot

NOMENCLATURE

c_p	specific heat at constant pressure
D	diameter of a cylinder
Gr	Grashof number $= g\beta(T_w - T_a)D^3/\nu^2$
h	heat transfer coefficient $= -k(dT/dr)_w/(T_w - T_a)$
k	conductivity of fluid
Nu	Nusselt number $= hD/k$
Pr	Prandtl number $= \mu c_p/k$
Re	Reynolds number $= UD/\nu$
T	absolute temperature
u, v	Cartesian components of the velocity
x, y	Cartesian coordinates.

Greek symbols

β	thermal expansion coefficient
ζ	curvilinear coordinate, $x^1 = \xi$, $x^2 = \eta$
μ	viscosity
ν	kinetic viscosity
ρ	density.

Subscripts

a	ambient value or air
m	values at the mean temperature
w	values on the wall.

cylinder correlations as Collis and Williams [1] ($T_m/T_a < 1.5$) poorly predict the heat transfer coefficient when extrapolated into the cold cylinder region; vice versa are those cold cylinder correlations of Zhukauskas and Ziugzda [2] and Amad [3] ($T_m/T_a = 0.63$–0.88).

The objective of the present study is to provide a unified correlation valid for both hot and cold cylinders. Also, a wide range of temperature ratios, which accounts for the property variation, is covered by the present correlation so that the significant influence of the temperature change on heat transfer can be properly predicted. Ultimately, a new unified correlation presented in this study would find practical applications to the hot-wire anemometry.

In the present study, main numerical results are obtained for a special case of $U = 22$ m s^{-1}, $D = 10$ μm. This means that the variation of Reynolds number was due solely to the change in physical properties with temperature ($T_a = 200$–900 K, $T_w = 300$–900 K). Out of the numerical results with this benchmark case, a new unified heat transfer correlation was established which is valid for $T_m/T_a = 0.67$–2.75 and $Re = 0.5$–40. Then, a large number of subsidiary runs were made to validate the new correlation by varying the magnitudes of the free-stream velocity and the cylinder diameter ($U = 11$–33 m s^{-1} and $D = 5$–25 μm). The validity of the present correlation was further assessed against a plethora of experimental data as well as other correlations currently in use.

2. NUMERICAL PROCEDURE

The governing equations (continuity, momentum and energy equations) are solved in an arbitrary curvilinear coordinate system, as in Kang *et al.* [4]. The buoyancy term is neglected ($Gr/Re^2 \simeq 10^{-6}$) and the radiation is also excluded (the ratio of the blackbody radiative to convective heat transfer is estimated to be less than 1%). The pressure work and dissipation terms are legitimately dropped from the energy equation since $Re \leqslant 40$. The thermophysical properties such as ρ, μ, c_p and k are considered to be functions of the temperature only, as given in Wang [5].

As shown in Fig. 1, no slip and constant temperature conditions are used at the cylinder surface. A uniform parallel flow ($u = U, v = 0, T = T_a$) is imposed at the upstream as well as at two horizontal boundaries. The first derivatives of u, v, T are taken to be zero far downstream from the cylinder. The size of the computation domain was chosen, as depicted in Fig. 1, after many trial runs with different locations of boundaries and mesh numbers. A nonuniformly spaced 125×51 grid system was selected and used for computation. In addition, the so-called C-type grid system is adopted so as to achieve high resolution especially near the stagnation point. It was found that the use of such a grid system was helpful in predicting the local heat transfer rate on the cylinder surface. Computation was performed over the full domain shown in Fig. 1, thereby leading to the confirmation of symmetries.

3. RESULTS AND DISCUSSION

Figure 2 displays a log–log scale distribution of calculated Nusselt numbers for various T_a and T_w values. It is evident that variation of fluid properties significantly affects the heat transfer rate from or to the cylinder surface. As the difference between T_a and T_w magnifies, a wider scattering is observed for Nu as a function of Re (evaluated at T_m) due to the effect of the property variation. In this regard, Fig. 2 dictates the

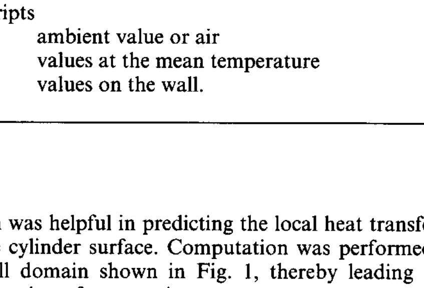

Fig. 1. A schematic of the computational model for the flow over a cylinder showing the domain of interest and the C-type grid system of 123×51 resolution.

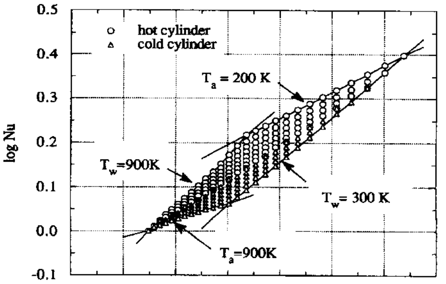

Fig. 2. The residual effect of temperature loading on the predicted Nusselt numbers for the case of $D = 10$ μm, $U = 22$ m s^{-1}.

residual effect of temperature loading and poses the need of correction factors.

In an attempt to derive a correlation for the scattered data shown in Fig. 2, two different temperature loading factors were considered and the results are presented in Fig. 3. For comparison, two available correlations of Collis and Williams [1] (for hot cylinders) and Amad [3] (for cold cylinders) are also shown in Fig. 3. Interestingly, when $p = -0.17$ was used, our data for a hot cylinder are consolidated into a single curve which deviates only slightly with the correlation of Collis and Williams [1]. However, there still exists a large scattering for the cold cylinder data. This suggests that the predicted Nusselt numbers from this study agree to within 5% with the experimental data of Collis and Williams [1] for the hot cylinder case, and that the correlation of Collis and Williams is unfortunately inadequate for the cold cylinder case. This is not surprising because their correlation was developed for a hot cylinder case. As for cold cylinder correlations, Amad [3] employed the viscosity ratio as a correction factor to account for the property variation. When converted into a temperature ratio, his correlation was found to have a temperature loading factor of $p = -0.26$. With this value, our calculated data for cold cylinders merge well into a single curve, but there still exists a wide margin from his correlation. Admittedly, such a large discrepancy might be caused from the assumptions employed here as well as the experimental uncertainties in ref. [3]. It is worthy of note that when extrapolated into the hot cylinder region the correlation of Amad deviates much from the present data and in turn from that of Collis and Williams [1].

From the above discussion, we are led to conclude that a new correlation needs to be developed which is valid for both hot and cold cylinders and is more accurate, especially for cold cylinder cases.

The next step is to derive a proper value of the temperature loading factor p; we made several hundreds of trials to find whether an optimum value of p is present in equation (1) by varying empirical coefficients and T_m/T_a values. But since all our efforts were fruitless, we came to suspect that the choice of temperature loading parameter T_m/T_a was not appropriate for the correlation. As an alternate to T_m/T_a, a new parameter T_w/T_a was considered and with this choice a unified correlation was successfully established:

$$Nu = (0.22+0.58Re^{0.45})\,(T_w/T_a)^{0.113}. \qquad (2)$$

In the above, the Reynolds number dependency to the power of 0.45 was retained which has been widely used in the literature (e.g. refs. [1, 3, 6]). Specifically, the empirical constants, 0.22 and 0.58, were obtained from the data of unity temperature loading, i.e. $T_w = T_a$. This new correlation and the predicted Nusselt numbers were drawn together in

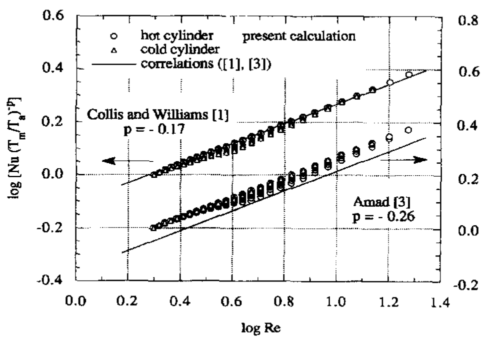

Fig. 3. Two available correlations for the Nusselt numbers and the present data adjusted with two different temperature loading factors for both the hot and cold cylinders ([1] $Nu(T_m/T_a)^{-0.17} = 0.24+0.56Re^{0.45}$, [3] $Nu(T_m/T_a)^{-0.26} = 0.207+0.497Re^{0.45}$).

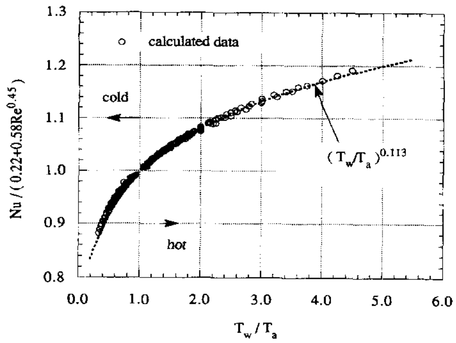

Fig. 4. A new unified correlation suggested in this work as a function of T_w/T_a.

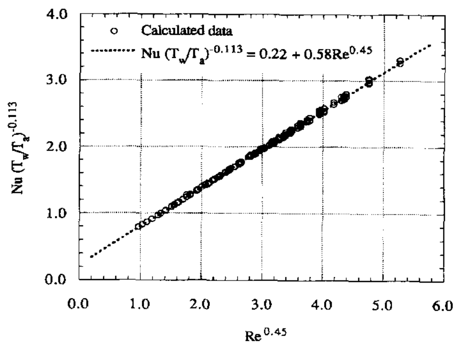

Fig. 5. Verification of the present unified correlation for both hot and cold cylinders against various cases with different values of free-stream velocity and cylinder diameter ($T_a = 200$–900 K, $T_w = 300$–900 K, $U = 11$–33 m s^{-1}, $D = 5$–25 μm).

Fig. 4. It is readily observable that our numerical results fit well with equation (2) for both hot and cold cylinders. However, since the correlation in equation (2) was based on the numerical results with $U = 22$ m s^{-1} and $D = 10$ μm, further validation runs were made over a wide range of free-stream velocity and cylinder diameter ($U = 11$–33 m s^{-1}, $D = 5$–25 μm), as shown in Fig. 5. It can be seen that the new correlation in equation (2) is also valid for other parameter values, thereby confirming its usefulness to practical situations relevant to hot-wire anemometry.

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The effect of thermofluid and geometrical parameters on convection of liquids through rectangular microchannels

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INTRODUCTION

The research into microscale flow and heat transfer phenomena conducted by Tuckermann and Pease [1, 2], Wu and Little [3, 4], Pfahler *et al.* [5, 6], Choi *et al.* [7] and Weisberg *et al.* [8] provided substantial experimental data and considerable evidence that the behavior of fluid flow and heat transfer in microchannels or microtubes without phase change may be substantially different from that which typically occurs in larger more conventionally sized channels and/or tubes. In an attempt to clarify some of the questions surrounding this issue, Peng and Wang [9, 10] and Peng *et al.* [11] recently investigated the heat transfer characteristics of liquid flowing through microchannel structures. In that work, the heat transfer and flow mode conversions for single-phase convection in microchannels, and the transitions induced by or associated with variations in the liquid thermophysical properties due to the increases in the liquid temperature through the heated microchannels, were studied. Wang and Peng [12] also studied the forced flow convection of liquid in microchannels both with and without phase change experimentally. It was found that fully-developed turbulent convection was initiated at Re = 1000–1500, and the heat transfer behavior in the laminar and transition regions was quite unusual and complicated.

In the present work, a series of experiments with several different microchannels were conducted to examine the single-phase convective heat transfer and better understand the fundamental physical phenomena associated with this type of flow situation. The physical fundamentals and nature of the flow and heat transfer phenomena in microchannels were examined to determine experimentally the influence of the liquid flow, thermal conditions and microchannel size on the flow and forced convective heat transfer characteristics for single-phase water flowing through microchannels.

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EXPERIMENTS

The test facility utilized in the current investigation and the experimental procedure have been described in detail by Wang and Peng [12]. The geometric parameters for the four different microchannels utilized are summarized in Table 1. Both water and methanol were employed as the working fluid and, for various tests, the temperature was varied from 11 to 28°C and 12–20°C (i.e. the liquid subcooling varied from 72 to 89°C and 45–53°C at ambient pressure) for water and methanol, respectively. The liquid velocities evaluated ranged from 0.2 to 2.1 m s^{-1} for water and 0.2–1.5 m s^{-1} for methanol.

As discussed previously, the applied surface heat flux of microchannel was calculated from the total input power as

$$q'' = \frac{Q}{N(2A_1 + A_2)} \tag{1}$$

Table 1. Geometric parameters of the test sections

Test	L [mm]	W [mm]	W_c [mm]	H_c [mm]
Section 1	45	18	0.8	0.7
Section 2	45	18	0.6	0.7
Section 3	45	18	0.4	0.7
Section 4	45	18	0.2	0.7